Pergamon

Int. J. Heat Mass Transfer. Vol. 41, Nos 4–5, pp. 809–810, 1998

0017–9310/98 $19.00+0.00

PII: S0017–9310(97)00177–4

TECHNICAL NOTES

Where did the Dittus and Boelter equation come from?

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(*Received* 12 *February* 1997 *and in final form* 4 *June* 1997)

INTRODUCTION

For turbulent flow in ducts perhaps the most widely quoted and used heat transfer correlation is that of Dittus and Boelter. At the start of my career in heat transfer over 25 years ago I was faced with a practical problem and a senior colleague said that what I needed to use was the Dittus and Boelter equation:

$$Nu = 0.023Re^{0.8}Pr^{0.4} \quad (1)$$

(although at that time we used Stanton number rather than Nusselt number).

Recently, I was consulted by a company who said that they had already tried the Dittus–Boelter equation and could I suggest something more accurate. There can be no doubt that equation (1) has a secure place among heat transfer engineers in industry.

This is easily confirmed by consulting the textbooks. A survey of 10 recently published general heat transfer textbooks found that nine of them quote equation (1), call it the Dittus and Boelter equation and refer to a University of California publication of 1930 [1] (also many of them pointed out that strictly the equation is for heating of the fluid and that for cooling of the fluid the power of the Prandtl number should be 0.3 instead of 0.4). The influence of the equation is confirmed by comments such as 'the most popular formula'. There is just one problem with the equation: it does not appear in the reference that is given for it.

THE TRUE DITTUS AND BOELTER EQUATIONS

Ignoring an obvious misprint in the paper (0.08 instead of 0.8) the equation in the original publication [1] is

$$\frac{Ud}{k} = 19.5\left(\frac{dV}{z}\right)^{0.8}\left(\frac{cz}{k}\right)^{n}$$

with $n = 0.3$ for cooling and 0.4 for heating. Properties are to be evaluated at the bulk temperature.

The meanings of the terms and their units are:

U heat transfer coefficient	[BTU/sq.ft/°F/h]
d inside diameter of tube	[in.]
k thermal conductivity	[BTU/sq.ft/h/°F/ft]
V mass velocity	[lb/sq.ft/s]
z absolute viscosity	[centipoises]
c specific heat	[BTU/lb/°F].

As the equation stands it is not dimensionless. Tidying up the British units so that they all are in terms of feet and hours gives

$$\frac{Ud\times 12}{k} = 19.5\left(\frac{(d\times 12)(V/3600)}{z}\right)^{0.8}\left(\frac{cz}{k}\right)^{n}.$$

Converting all the variables to basic SI units (or any other consistent set of units) gives

$$\frac{Ud}{k} = 0.0241\left(\frac{dV}{z}\right)^{0.8}\left(\frac{cz}{k}\right)^{0.4}$$

for heating of the fluid, and

$$\frac{Ud}{k} = 0.0264\left(\frac{dV}{z}\right)^{0.8}\left(\frac{cz}{k}\right)^{0.3}$$

for cooling of the fluid.

These are essentially the same as the Dittus and Boelter equations given [2–4] shortly after the original paper was published:

$$Nu = 0.0243Re^{0.8}Pr^{0.4} \quad (2a)$$

for heating of the fluid, and

$$Nu = 0.0265Re^{0.8}Pr^{0.3} \quad (2b)$$

for cooling.

Many early books and journals correctly give this version of the Dittus and Boelter equations. More recently it has appeared much less frequently, and only in more specialised texts. Sometimes the constants are simplified to 0.024 and 0.026.

An obvious reason for this confusion is that the original reference, [1], is not easy to obtain. This does not now pose a problem. A facsimile, complete with original printing mistakes, has been published [5].

ORIGIN OF $Nu = 0.023Re^{0.8}Pr^{0.4}$

McAdams [6], in the 2nd edition of his book, gives this equation, with fluid properties at the bulk temperature. He does not give a clear reference. In fact he refers to the 1st edition of his book, as though the equation appears there, but it does not.

Confusion seems to have arisen because McAdams has simplified an equation,

$$Nu = 0.0225Re^{0.8}Pr^{0.4}$$

that does appear in the earlier edition, and does appear to be an equation that McAdams himself had fitted to data. Certainly the difference between 0.0225 and 0.023 is not significant given the spread of the data. That he did not regard this as a significant change is clear from [7] where he refers to a constant in the equation of 0.023 and gives a paper by Ullock and Badger [8] as a reference. However the paper by Ullock and Badger in fact gives the constant as 0.0225 and refers to McAdams' book [4] as the source!

So it may be that the earliest generally accessible reference to the equation, with a constant of 0.023, is the 2nd edition of McAdams' book in 1942, and the earliest reference of any sort, though in a curious roundabout way, and not being recommended as a general equation, is in McAdams' paper of 1940 [7]. There can be little doubt that equation (1) originated with McAdams, but he did not do a very good job of publicising the fact.

The remaining question is when the wrong reference for $Nu = 0.023Re^{0.8}Pr^{0.4}$ started to be used, i.e. when did this equation start to be attributed to Dittus and Boelter in 1930 [1]?

Certainly there was a tendency very early on for any equation of the form of equations (1) or (2) to be called the Dittus and Boelter equation. In 1934 [13]

$$Nu = \text{constant}\ Re^{m}Pr^{n} \tag{3}$$

is specifically referred to as being 'usually known as the Dittus and Boelter equation'. Many equations had been suggested by this time. The contribution of Dittus and Boelter was important in the use of properties at the bulk fluid temperature, which is often more convenient in practice than the use of the film temperature.

One possible explanation is that there seems to have been a change over time where the phrase Dittus and Boelter moved progressively from meaning equation (2), to meaning equation (3) and then to meaning the best equation of the form of equation (3). As already mentioned, the best equation of the form of equation (3) is equation (1), $Nu = 0.023Re^{0.8}Pr^{0.4}$, apparently introduced by McAdams in 1942. It is difficult to say when the earliest incorrect linking between equation (1) and ref. [1] might have been published but it had certainly occurred by 1954 [9].

Perhaps a more plausible explanation is linked with the confusion over whether the constant in equation (1) should be 0.0225 or 0.023. As already mentioned McAdams did not consider the change from one to the other to be significant. If we accept that $Nu = 0.0225Re^{0.8}Pr^{0.4}$ is simply an early version of equation (1), then the incorrect description of it as the Dittus and Boelter equation occurred as early as 1937 [10]. At least Ullock and Badger, the authors of [10], who presumably had not seen the original Dittus and Boelter paper, did not make the mistake of giving that paper as the reference (an example a number of more recent authors might follow to advantage!). Instead, presumably through a misreading of the relevant section, they gave McAdams [4] as the source of the information.

The question of the accuracy of equation (1) is not the subject of this paper, but [11] suggests that it should continue to be used for approximate calculations (and there is no point in using the alternative version; equation (1) can be used for either heating or cooling) and that the Gnielinski [12] equations may be used for greater accuracy.

CONCLUSION

It is obviously too late now to change the description of $Nu = 0.023Re^{0.8}Pr^{0.4}$ as the Dittus and Boelter equation. However, ref. [1] is wrong. It would be better to say the Dittus and Boelter equation, as introduced by McAdams [6].

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